THE BAROMETRICAL DETERMINATION OF HEIGHTS

A PRACTICAL METHOD

OF

BAROMETRICAL LEVELLING AND HYPSOMETRY

FOR

SURVEYORS AND MOUNTAIN CLIMBERS

BY

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NEW YORK
SPON & CHAMBERLAIN, 12 CORTLANDT STREET
LONDON
E. & F. N. SPON, LIMITED, 125 STRAND
1898

W. M.

TROW DIRECTORY
PRINTING AND BOOKBINDING COMPANY
NEW YORK

PREFACE.

THE discrepancies arising in the calculation of mountain heights by the barometrical formulæ which have hitherto been in use have brought this valuable and in many cases only applicable method into disrepute. The fault has lain in the formulæ, not in the method, which is one susceptible of great accuracy. These formulæ have either been based upon unwarrantable assumptions or have failed to take account of all the conditions obtaining in the problem.

The present essay was originally entered in the Hodgkin Prize Competition, held under the auspices of the Smithsonian Institution, and was awarded honorable mention. In it the important problem of Barometrical Hypsometry, which has not been touched upon since 1851, when it was discussed by Guyot, has been gone over anew and brought up to date. Important errors in the older formulæ have been detected and a new method has been furnished which is rigidly accurate in theory and which in practice will give reliable results under all conditions.

F. J. B. C.

THE BAROMETRICAL DETERMINATION OF HEIGHTS.

ONE of the most important applications of the known properties of air has been a deduction from them of a means of finding the vertical height between any two points, and the problem of measuring the vertical distances between any two levels is one that has engaged the attention of a number of mathematicians and physicists for many years.

Laplace, in the "Mécanique Céleste," gave what at the time was considered a complete solution of the problem; but as it was based upon several unwarrantable assumptions, and took no account of the aqueous vapor in the atmosphere, it was at best an approximation.

The complete formulæ as given by him is

$$Z = \log \frac{h}{H} \, 18336 \begin{cases} (1 + 2\,\dfrac{(t + t')}{1000} \\ (1 + .0028371 \cos.\ 2\,L \\ (1 + \dfrac{(\log \frac{h}{H} + .868589)\frac{Z}{a}}{\log \frac{h}{H}} \end{cases}$$

where $Z =$ the difference of level in metres;
$a =$ Earth's mean radius $= 6,366,200$ metres;
$L =$ mean latitude of the two stations.

And further

$$\text{At station} \begin{cases} \text{Lower} \begin{cases} h = \text{height of barometer;} \\ T = \text{temperature of barometer;} \\ t = \text{temperature of air;} \end{cases} \\ \text{Upper} \begin{cases} h' = \text{height of barometer;} \\ T' = \text{temperature of barometer;} \\ t' = \text{temperature of air;} \end{cases} \end{cases}$$

$$\text{and } H = h + h'\left(\frac{T - T'}{6196}\right).$$

The first parenthesis in the terminal factor is the correction for the difference of temperature of the two levels. It assumes that the problem would be the same if the air between the two levels were of a uniform temperature—the mean of what is observed at the two levels. As a matter of fact, if the two stations are remote, a large range of temperatures may be found at intervening points.

The second parenthesis is the correction for the change of gravity with the latitude. It assumes that gravity increases regularly according to a law as we go from the equator to the poles—a supposition which we now know to be true only in a general way. The third parenthesis is the correction for the decrease of gravity in a vertical direction. It is based upon the Newtonian law that externally to the earth's surface gravity decreases inversely as the square of the distance from the centre of mass. From careful pendulum experiments we know that such a law does not hold near the earth's surface, large masses of matter in different localities causing variations that are not to be accounted for by any simple law.

Baily, in his "Astronomical Tables and Formulæ," gives the following formula:

$$\chi = 60345.51 \left\{ 1 + .0011111\,(t + t' - 64^\circ) \right\}$$

$$\times \log \left\{ \frac{\beta}{\beta'} \times \frac{1}{1 \times .0001\,(\tau - \tau')} \right\} \times \left\{ 1 + .002695 \cos. 2\phi \right\}$$

where ϕ = latitude;

$\left.\begin{array}{l} \beta = \text{height of barometer;} \\ \tau = \text{temperature of mercury;} \\ t = \text{temperature of air;} \end{array}\right\}$ at lower station.

$\left.\begin{array}{l} \beta' = \text{height of barometer;} \\ \tau' = \text{temperature of mercury;} \\ t' = \text{temperature of air;} \end{array}\right\}$ at upper station.

Feet, inches, and the Fahrenheit scale are here used. Here the same assumption is made in re-

gard to the increase of gravity with the latitude as in Laplace's formula, and no account is taken of the moisture in the air.

Bessel first introduced in his formula, *Astronomische Nachrichten*, No. 356, a separate correction for the effects of moisture. Laplace's barometrical coefficient is retained, but the correction for change of gravity is considerably modified.

Elie Ritter in his formula, "Mémoires de la Société de Physique de Genéve," tome XIII., p. 343, gave a correction for moisture. The values of the barometrical and thermometrical coefficients are derived from Regnault's determinations, and a new method is proposed for applying the correction due to the expansion of the air, which is made proportional to the square of the differences between the observed temperatures at each station.

Baeyer's formula, Poggendorf's *Annalen der Physik und Chemie*, tome XCVIII., p. 371, does not belong to either of the two classes just mentioned; for while it keeps Laplace's barometrical and thermometrical coefficients, it corrects the effects of temperature by a method analogous to that of Ritter, and it entirely neglects the effect of aqueous vapor.

Plantamour in his tables substitutes for Laplace's barometrical coefficient that derived from the probably more accurate determination of the relative weight of air and mercury by Regnault,

viz., 18404.8 metres. Laplace used the results of Biot and Arago, and the coefficient deduced from it was 18317 metres. This coefficient was, however, empirically increased to 18336 metres in order to adjust the results of the formula to those furnished by the careful trigonometrical measurements made by Ramond for the purpose of testing its correctness.

An error in all these formulæ was the assumption of an absolutely invariable barometric coefficient. The barometric coefficient in Baily's formula, 60345.51, varies under different conditions of pressure, temperature, and relative humidity, from 55,000 to 65,000, and in fact the chief thing to do before applying any formula is to compute exactly the barometric coefficient. The barometric coefficient, 18336 metres, which was substituted in Laplace's formula in order to make the data of some observations conform to Ramond's trigonometrical determinations, would have had to be changed if other observations had been taken under different conditions of the atmosphere.

We shall demonstrate this in what follows, elucidating the problem at each step, so that it may be easily understood by any reader having a slight knowledge of mathematics.

If the earth were a mere level plane, the vertical distance above it of any point would be a simple matter to determine ; but the earth is not a sphere, nor a spheroid. Strictly it is only ap-

proximately a regular figure. If the sea were at rest, a figure very nearly corresponding to its surface would be an ellipsoid of revolution, having an equatorial semi-diameter of 20,926,200 feet, and a polar semi-diameter of 20,854,900 feet, giving an ellipticity of $\frac{1}{293,465}$.*

From a comparison of the different measured arcs of meridians, Colonel Clarke found that the surface most nearly agreeing with the sea-level is an ellipsoid (not of revolution) having for its equatorial section an ellipse with a major sėmi-axis at 8° W. lon. of 20,926,629 feet, and a minor semi-axis of 20,854,477 feet.

If the air also were at perfect rest it would distribute itself about the earth under the influence of gravity, so that we could trace out in it surfaces of equal density, or in fact equipotential surfaces, and these surfaces would be more or less similar and similarly situated to the surface of the earth as just considered. In the strictest sense, then, the height of a point above the surface of the earth would be the perpendicular distance between the standard equipotential surface and the point. Practically, however, in measuring heights, we determine the perpendicular distance between the equipotential surface passing through the upper point and that passing through some lower point of reference.

* Colonel Clarke, Geodesy, p. 319.

In measuring heights we have three methods at our disposal :

I. That by levelling.

II. That by vertical angles.

III. That by weighing a vertical column of air between the levels, *i.e.*, the barometric method.

Without entering into a discussion of the comparative merits of the different systems, it may be stated that in determining mountain heights with any degree of accuracy, the barometric method is usually the only practicable one. In determining the height of a mountain like Chimborazo, as Mr. Whymper did, it is wholly out of the question to level it, and vertical angles taken at great distances can only lead to an approximation.

The determination of a height by a barometer is equivalent to the weighing of a column of air under varying conditions. Reduced to its simplest terms the atmosphere is supposed to be absolutely at rest, of a uniform temperature throughout, absolutely dry, and the value of g—the force of gravity at the place—to be known. Then it will be easy to obtain the weight of a vertical column of air between any two points, and thereby the height of this column.

By Mariotte's law we know that the density of air is proportional to the pressure to which it is subjected. If we know accurately the weight of each cubic foot of air in the column, the pressure of the whole column per square foot will be ar-

rived at by simply adding the individual weights. Such a summation, when the elements vary according to a law, can be easily effected by the integral calculus.

Let the weight of a cubic foot of air which is the increment or increase of pressure for every foot that we descend be represented by δp. Now this weight or increment is itself proportional to the pressure by Mariotte's law and also proportional to the height of the element, which is here supposed to be the unit of measure—one foot. Represent this element of height by δh, then $\delta p = c.p.\ \delta h$, where c is some constant.

Or $\delta h = k\ \dfrac{\delta p}{p}\cdot$ Summing our elements, or in other words, integrating, we have

$$\sum_{h=o}^{h=x} \delta h = k \sum_{p_x}^{p_o} \frac{\delta p}{p} = k \log \frac{p_o}{p_x}$$

$\therefore\ h = k \log \dfrac{p_o}{p_x}$ where p_o is the pressure given by the barometer at the lower station, p_x the pressure at the upper station and h is the height.

If we let $h = 1$, then $k = \dfrac{1}{\log \dfrac{p_o}{p_1}} = \dfrac{1}{\log \dfrac{p_1 + w}{p_1}}$

where w represents the weight of a cubic foot of air, expressed, of course, in the same terms as

the pressure. If then we determine once for all the weight of a cubic foot of air at the pressure p_1 for the temperature t and value g of gravity, we have a value of k which can be used in all calculations where the conditions remain the same. But the above formula, $h = k \log \frac{p_0}{p_x}$ holds only provided the conditions remain the same throughout the whole column of air—a state of affairs that never occurs in nature except for narrow limits. In ascending from one level to another we pass through many varying conditions. One layer of the atmosphere may be much warmer than another immediately above or below it.

The relative humidity varies from point to point and from hour to hour. The value of g does not vary greatly for different levels at the same place, but varies sufficiently at places widely removed from each other on the earth's surface to be taken into account. Violent commotions of the atmosphere are another disturbing factor on the pressure, the swirls and eddies of a storm often causing the barometer to jump up and down a hundredth of an inch or more. Under these conditions the attempt to weigh a long column of air—to determine a great height—by taking conditions (pressure, temperature, and relative humidity) at the base and summit of a mountain simultaneously, and then taking the mean of the latter two is futile, although baromet-

rical formulæ have usually been applied in this manner.

There remains, however, a means by which a very close estimate can be arrived at in spite of all these obstacles. That is to take the conditions (t, p, and r.h.) at frequent intervals, so that we can be reasonably sure that from one point to another the conditions of our original formula hold to a close approximation.

The occasions are very rare, especially when the weather is at all settled, where any very sudden changes take place for an ascent of 500 feet or more. But even then it is easy to multiply stations and take the readings of the barometer, thermometer, and psychrometer. Our total height will then be the algebraic sum of our elements. As a correction we can take readings again on descending and average the heights by ascent and descent.

All this necessitates the separate determination of the constant k for each interval of ascent, and this requires that besides knowing the extreme pressures we also compute the weight of a cubic foot of air as found either at the top or bottom of the interval.

The mercurial barometer is the only instrument at the present time that we can rely upon for taking pressures. The aneroid barometer is a misnomer; it should be called the aneroid baroscope. The best of these instruments are entirely unreliable and irregular in their action. Mr.

Whymper ("How to Use the Aneroid Barometer") has done an important service in showing conclusively that they are worse than useless, as they always underread the mercurial barometer, and in a most irregular way at that.

The mercurial barometer should be suspended from a tripod and exposed to the atmosphere for about ten minutes, when several readings should be taken, and the average entered as the reading for the station at that time. The readings of the thermometer and psychrometer are taken simultaneously. The height of the mercurial column is carefully reduced afterward to freezing, and the correction for capillarity is applied. Where great accuracy is aimed at, or where there is reason to believe that the value of g at the place differs considerably from the standard value—that of Paris, for our purposes, since most of our data were established at that place by Regnault—we must apply this correction also.

Besides a barometer two other instruments only are required—a thermometer and a hygrometer. The thermometer attached to the barometer is sufficient to determine the temperature of the atmosphere.

Regnault found that the weight of a litre of dry air at Paris under a pressure of 760 mm. of mercury was 1.293233 grammes. This figure is the base upon which all our calculations rest. The force of gravity at Paris being g_p—32.1747 foot-seconds, or 980.94 dynes—the standard of height

of our barometer, 760 mm., will vary with the force of gravity. Consequently, the weight of a litre of dry air at any other place x will be

$$1.293233 \frac{g_x}{g_p} \text{ grammes.}$$

[1.293233 grammes = 19.95712 grains Troy.]

The value of g has been determined with a considerable degree of accuracy at various points of the earth's surface. Where the value has been determined near some place where the altitude is to be determined, it is best to rely upon that value. Laplace's and the other formulæ assume that this value varies according to a regular law with the latitude, and also with the altitude above the sea-level. Actually such a law exists only to a very limited extent.

While it is probable that the decrease of gravity as we ascend directly into the air (*i.e.*, by balloon), would conform more or less nearly to the law of the inverse squares, such a law is by no means the case on a mountain-slope. Mendenhall found that the value of g on the summit of Fuji-yama was greater than at the base. On many mountains it may be slightly less at the top, but wanting definite measurements it would be safer to assume that there is no appreciable change.

The following table will give some idea of the variability of g.

TABLE I.

VARIABILITY OF GRAVITY AT:

Greenwich,	$g = 32.1912$	feet — seconds.
Paris,	$g = 32.1747$	
Washington,	$g = 980.100$	dynes.
Mt. Hamilton, Cal.,	$g = 979.651$	
Honolulu, H. I.,	$g = 978.936$	
Waikiki, H. I.,	$g = 978.922$	
Kawaihæ, H. I.,	$g = 978.803$	
Kalaieha, H. I.,	$g = 978.490$	— 6660 ft.
Mauna Kea, H. I.,	$g = 978.060$	— summit.

The last three determinations are for the slopes of Mauna Kea, the first, Kawaihæ, at the base, the second, Kalaieha, half way up, and the third at the summit. In general, the value of g is less on islands than on continents.

Owing to the ellipticity of the earth and the effect of centrifugal force becoming less as we approach the poles, gravity tends to increase from the equator toward the poles. Let the elliptic section through the poles be represented by

$$\frac{r^2 \cos.^2\alpha}{a^2} + \frac{r^2 \sin.^2\alpha}{b^2} = 1 \therefore r^2 = \frac{a^2 b^2}{b^2 + c^2 \sin.^2\alpha}$$

By the law of inverse squares,

$$g = \frac{k}{r^2} \therefore g = \frac{k}{a^2 b^2}(b^2 + c^2 \sin.^2\alpha) = k' + k'' \sin.^2\alpha$$

Therefore, the increment of g from the equator to the poles is proportional to the square of the sine of the latitude. Considering the effect of centrifugal force we have C. F $=$ K cos. α. Resolving this in the direction of gravity its result-

ant becomes K $\cos.^2 a$ $\therefore$ g (the resultant force of gravity) = f (actual force) − K $(1 - \sin.^2 a)$ $\therefore$ g $= K' + K'' \sin.^2 a$.

Thus the increase of g from diminution of the centrifugal force varies according to the same law and we can combine the formulæ thus: $g = g_e (1 + .005133 \sin.^2 L)$, where g_e is the force of gravity at the equator. An approximate value of g_e is 32.088, but, as we have seen, it is not constant. The formula given above, with the coefficient .005133, was deduced originally by Clairaut. Measurements show that gravity conforms to a certain degree with this law, and it is advisable where definite determinations are wanting to apply such a correction. It must be borne in mind, however, that gravity may be less on an island situated far to the north than at a continental point near the equator.

The weight of a litre of pure mercury Regnault found to be 13596 grammes. Let us suppose that at Paris, the air being perfectly dry, the temperature is 0° Cent. and the pressure p expressed in inches of mercury. By our barometrical formula

$k = \dfrac{1}{\log \dfrac{p + w}{p}}$, where w is the weight of a cubic foot of air expressed in inches of mercury. Therefore,

$$k = \log\left(\frac{1}{\dfrac{p + \dfrac{1.293233}{13596} \times 12}{p}}\right)$$

But suppose the temperature and relative humidity are not zero. As the temperature increases the weight of a cubic foot of air becomes less for the same pressure. The coefficient of expansion of air and most gases is .00367 per degree Cent.

$$\therefore V_t = \frac{V_o}{1 + \alpha_t} \therefore V_t = \frac{V_o}{1 + .00367_t}$$

The relative humidity also affects the weight of a given volume of air. Moist air is always lighter than dry air. The relative humidity is defined to be the ratio of the pressure of the vapor in a given volume of air to the pressure of the vapor in the same volume at saturation.

Let V denote a volume of air, H its pressure, f the pressure of the vapor in it and t the temperature. The entire gaseous mass may be divided into two parts, a volume V of dry air at temperature t and pressure H—f, the weight of which is

$V \times 1.293233 \times \frac{1}{1 + \alpha t} \times \frac{H - f}{760}$, and a volume V of aqueous vapor at the temperature t and pressure f. Since the density of aqueous vapor is $\frac{5}{8}$ that of dry air, the weight of this latter mass is

$$\tfrac{5}{8} V \times 1.293233 \times \frac{1}{1 + a t} \times \frac{f}{760} \cdot$$

The sum of these two weights is the weight required, viz.: $V \times 1.293233 \times \frac{1}{1 + \alpha t} \times \frac{H - \frac{3}{8} f}{760}$.

The pressures in mm. of mercury at saturation for different temperatures of aqueous vapor have been carefully determined by Regnault and are given in the following table :

TABLE II.

PRESSURES IN MM. OF MERCURY AT SATURATION FOR DIFFERENT TEMPERATURES OF AQUEOUS VAPOR.

Temperatures, Cent.	Force of vapor, mm.	Temperatures, Cent.	Force of vapor, mm.
— 32	0.32	10	9.17
— 20	.93	15	12.70
— 10	2.09	20	17.39
— 5	3.11	25	23.55
0	4.60	30	31.55
5	6.53	40	54.91

A more complete table is given at the end.

The force of the vapor in the atmosphere is obtained by observing the dew-point, which gives us the temperature at which saturation occurs for the given pressure. The actual force of the vapor will be the tension of saturated vapor at the dew-point. An efficient instrument for determining the dew-point is Dines's Hygrometer. Or the relative humidity may be obtained from tables by readings of the wet and dry bulb hygrometer. The results of this latter instrument, sometimes known as August's Psychrometer, are far from accurate.

We can thus, from the foregoing formula, determine the weight of a cubic foot of moist air

under any given conditions of temperature, pressure, and humidity.

At the pressure p, reduced to Paris, temperature t and vapor tension f, the weight of a cubic foot of moist air, expressed in inches of mercury, is

$$1.293233 \times \frac{12}{13596} \times \frac{1}{1+.00367t} \times \frac{p-\frac{3}{8}f}{29.9212} = \frac{p-\frac{3}{8}f}{1+.00367t} \times .0000381468.$$

$$k = \frac{1}{\log\left(\dfrac{p + \dfrac{p-\frac{3}{8}f}{1+.00367t} \times .0000381468}{p}\right)}$$

where t is Centigrade and p expressed in inches of mercury reduced to Paris and freezing, and corrected for capillarity. The corrections for capillarity and freezing are taken from carefully prepared tables which are easily obtained. The key, then, to all our barometric determinations is the computation of the coefficient k. Having obtained this for a range where the variations of this factor are inappreciable, it remains only to get the height by applying the formula

$$h = k \log \frac{p_0}{p_x}.$$

The computations are made after the ascent from the data collected. These can be taken without much trouble during the ascent and descent. The readings should be carefully recorded in a

systematic manner, so that no doubt can appertain to them. As a rule, one man should carry the barometer or barometers, and nothing else. He should be intrusted with the readings of the mercurial column and the thermometer attached. Another man should carry and record, simultaneously, the hygrometer. At the point of departure—the station from which the heights are to be estimated—the tripod should be set up, the barometer suspended, and after exposing it to the air for ten minutes or more, to make sure that the thermometer attached and all the parts of the barometer are at the same temperature as the air, the readings should be taken. At the same time the hygrometer should be read. The body of the observer should not be too close to the instruments. It is well to take several readings at each station and average them. Whenever in the ascent (or descent) it becomes evident that the conditions are changing, a stop should be made, station noted, and readings taken. Whether the conditions are changing or not can be determined from time to time without a stop by simply noting the temperature or glancing at the wet and dry bulbs, if a psychrometer is taken. Whenever a prolonged stop is made, as at dinner or on camping for the night, the readings should be taken directly after the halt and again on starting out, taking this station as a new point of departure. It is possible that a long ascent may be made without a perceptible change of conditions,

and in this way tedious computations may be avoided.

Such is the problem of measuring heights by the barometer—a process perfectly intelligible at every step, and consisting of weighing every layer of the atmosphere through which the ascent is made.

Since in a mixture of one or more gases in a given space, each gas behaves dynamically the same as if it occupied the space alone, it is evident that where the conditions remain the same, we can compute heights by three different sets of data. We can consider the air as a mixture of dry air and vapor, as has been done in the foregoing discussion, using the formula $h = k \log \frac{p_o}{p_x}$ where k is derived from the weight of a cubic foot of moist air under the obtaining conditions, and p_o and p_x are the actual barometric pressures observed. Or we may weigh a column of dry air between the two stations, using the formula $h = k' \log \frac{p_o - f_o}{p_x - f_x}$ where p_o and p_x are the barometric readings, and f_o f_x are the vapor tensions at the extreme stations, while k' is derived from the weight of a cubic foot of dry air at pressure $p_o - f_o$ and temperature t.

Again, making use of the vapor alone, we can employ the formula $h = k'' \log \frac{f_o}{f_x}$, where k'' is derived from the weight of a cubic foot of vapor at the pressure f_o and temperature t. The last two

methods will not be so reliable as the chief method, since the tension f, which is derived from the dew-point, cannot be measured so closely as the pressure p by a good barometer. Still, all these methods may be employed as checks on the others.

A practical example is appended illustrating the method in practice and showing within what limits of accuracy heights can be thus determined. A seven-place table of logarithms will be advisable for the computations.

Barometer at lower station read 30.2784, and at upper station 30.224, corrected for gravity, but not for capillarity and temperature. Temperature at both stations 15° C., and f at lower station was .2434 inch. The error of the barometer was very slight, if any.

$$
\begin{array}{rl}
 & 30.2784 \\
+ & \underline{.085} \text{ correction for freezing point.} \\
 & 30.1934 \\
+ & \underline{.037} \text{ correction for capillarity.} \\
 & 30.2304 \text{ corrected reading for lower station.}
\end{array}
$$

$$
\begin{array}{rl}
 & 30.224 \\
- & \underline{.085} \text{ correction for freezing point.} \\
 & 30.139 \\
+ & \underline{.037} \text{ correction for capillarity.} \\
 & 30.176 \text{ corrected reading for upper station.}
\end{array}
$$

$\frac{3}{8}\,f = .091$ inch.

$\therefore\ p - \frac{3}{8}\,f = 30\ 176 - .091 = 30.085$

$1 + .00367\,t = 1.055$

$\log 30.085 = 1.4783500$

$\log 1.055 = 0.0232525$

$\log .0000381468 = \bar{5}.5814527$

$$\therefore \log \frac{p - \frac{3}{8}\,f}{1 + .00367\,t} \times .0000381468 = \bar{3}.0365502$$

This logarithm corresponds to .001087 inch. This shows that under the conditions a cubic foot of air weighed .001087 inch of mercury.

$$\begin{aligned} p + .001087 &= 30.1771 - \\ \log 30.1771 &= 1.4796775 \\ \log 30.176 &= \underline{1.4796617} \\ & \quad .0000158 \end{aligned}$$

$$\therefore \frac{1}{.0000158} = 63291.1 = k$$

$$\begin{aligned} \log p_1 &= 1.4804438 \\ \log p_2 &= \underline{1.4796617} \\ & \quad .0007821 \times k = 49.499 + \end{aligned}$$

It will be noticed that the last four logarithms, though actually natural logarithms in the formulæ, are here computed as common logarithms, since the modulus cancels out.

Therefore the height between the two stations is by the computations 49.5 feet. The height as previously determined by levelling was 50 feet. A number of determinations of the same height were made under widely differing conditions of the atmosphere and the results all came out within half a foot of the correct height, 50 feet—some of them being remarkably close.

Such short distances are very severe tests for a barometer. Where the heights are greater the proportional error will be less. By no refinement is it possible to measure with a barometer much less than a foot, since, as we have seen, the weight of a cubic foot of air is equivalent to only .001 (+ or −) inch of mercury, and our barometers do not read finer than thousandths. By careful

work it is probable that our error need never exceed 5 feet in 1000. By our method also of multiple stations, the errors, which are as liable to be above as below, will be apt to eliminate themselves in the summation.

The altitude of Mont Blanc was computed by Delcros, from measurements taken by MM. Bravais and Martins, to be 4810 metres, which came strikingly close to the result obtained by a geodetic survey, viz.: 4809.6. But such a coincidence must not deceive us. It is a mere coincidence, as Delcros's formula is not susceptible of such accuracy—particularly as it wholly neglects the moisture of the air.

TABLE III.

Elastic Force of Saturated Aqueous Vapor. Expressed in inches of Mercury and Fahrenheit Temperatures.

Temp.	Force.	Temp.	Force.	Temp.	Force.
− 31	.0087	13	.0783	57	.4655
− 30	.0092	14	.0818	58	.4825
− 29	.0098	15	.0857	59	.5000
− 28	.0104	16	.0898	60	.5179
− 27	.0110	17	.0940	61	.5365
− 26	.0117	18	.0984	62	.5558
− 25	.0124	19	.1030	63	.5756
− 24	.0131	20	.1078	64	.5962
− 23	.0138	21	.1128	65	.6173
− 22	.0146	22	.1179	66	.6392
− 21	.0154	23	.1233	67	.6616
− 20	.0163	24	.1289	68	.6847
− 19	.0171	25	.1347	69	.7084
− 18	.0181	26	.1407	70	.7329
− 17	.0190	27	.1469	71	.7583
− 16	.0200	28	.1534	72	.7844
− 15	.0210	29	.1600	73	.8113
− 14	.0221	30	.1668	74	.8391
− 13	.0232	31	.1739	75	.8676
− 12	.0244	32	.1811	76	.8970
− 11	.0257	33	.1883	77	.9272
− 10	.0270	34	.1959	78	.9582
− 9	.0283	35	.2037	79	.9902
− 8	.0297	36	.2119	80	1.0232
− 7	.0312	37	.2204	81	1.0572
− 6	.0327	38	.2291	82	1.0922
− 5	.0343	39	.2382	83	1.1281
− 4	.0359	40	.2476	84	1.1651
− 3	.0376	41	.2572	85	1.2031
− 2	.0395	42	.2672	86	1.2421
− 1	.0454	43	.2775	87	1.2821
0	.0434	44	.2882	88	1.3234
1	.0454	45	.2993	89	1.3659
2	.0476	46	.3108	90	1.4097
3	.0498	47	.3228	91	1.4546
4	.0521	48	.3351	92	1.5008
5	.0545	49	.3477	93	1.5482
6	.0570	50	.3608	94	1.5969
7	.0597	51	.3743	95	1.6468
8	.0625	52	.3882	96	1.6980
9	.0654	53	.4027	97	1.7508
10	.0684	54	.4176	98	1.8050
11	.0716	55	.4331	99	1.8607
12	.0749	56	.4490	100	1.9179

APPENDIX ON THE AIR BAROMETER.

THE mercurial barometer as originally invented by Toricelli has remained up to the present day practically the only instrument by which pressures of the atmosphere are measured. It is evident that instead of balancing the pressure of the atmosphere by a column of mercury, we can do the same thing by opposing the pressure of a given volume of confined air. There are many advantages in the latter method over the former. The chief reason why air barometers have not been employed is that the effect of temperature superadded to that of pressure causes such wide deviations from the normal readings that corrections must be applied before a result can be obtained. But for accurate determinations we have seen that a temperature correction is equally essential in a mercurial barometer, so that this apparent advantage vanishes.

An efficient air barometer should be constructed as follows: A thermometer tube, with a properly proportioned bulb and calibre, should be wound in the form of a spiral with the bulb in the centre. It should be of comparatively great length and the bore of tolerably uniform calibre, else it must be calibrated. It should be filled with dry air, and at a temperature and pressure causing an ex-

pansion greater than it is ever likely to be subjected to in practice, and should be plugged with a small index of mercury. This coil should be securely fastened in a box divided into two parts—the upper smaller part just large enough to contain the coil and a sensitive thermometer, the lower part capable of containing pieces of ice and fitted with two tubes by which water can circulate throughout both partitions. The upper surface of the box should consist of a sheet of thick glass through which the barometer and thermometer can be read, and the end of the barometer tube should project outside of the box. It will thus be seen that the barometer can be read at any time surrounded by air of a given temperature, or surrounded by flowing ice-water. In one case the readings will have to be reduced to zero, in the other they are already reduced to zero.

In using such a barometer it will be seen that if V is the reading at a lower station and v the reading at an upper, the height h will be given by the formula $h = k \log. \frac{v}{V}$ (1), without further corrections, provided the range between V and v is at a uniform temperature.

k here is as before the barometrical coefficient which must be carefully calculated.

We have already

$$\frac{1}{k} = \log \left(1 + \frac{1 - \frac{3}{8} \frac{f}{P}}{1 + .00367\,t} \times .0000381468 \right)$$

where f is the pressure of the aqueous vapor and P the total pressure at a station.

Now let us suppose determined once for all the reading V_0 which the barometer assumes at zero degrees and a pressure equal to 760 mm. of mercury at Paris.

f is the pressure of saturated aqueous vapor at the dew-point, which must be determined, and t is the temperature of the range. Let us suppose that having determined the dew-point we find from tables that the pressure f is a fraction β of the standard pressure, viz., the weight of a column of mercury 760 mm. high at Paris. Reckoning our temperatures according to the "absolute scale," we have

$P_0 = \frac{C.\ T_0}{V_0}$ & $P = \frac{C.\ T}{V}$, where P_0 and P represent the standard pressure and the pressure at the station, V_0 and V are the readings of the barometer under standard conditions and at the station, $T_0 = 273$ and $T = 273 + t$ the temperature of the range.

$$\therefore \frac{f}{P} = \frac{\beta P_0}{P} = \beta . \frac{T_0}{T} . \frac{V}{V_0}$$

$$\therefore \frac{1}{k} = \log \left(1 + \frac{1 - \frac{3}{8}\beta \frac{273}{(273 + t)} . \frac{V}{V_0}}{1 + .00367\,t} \times c. \right)$$

where $c = .0000381468$.